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Book Review: *Random Walks and Random Environments, Volume 1: Random Walks*

Random Walks and Random Environments, Volume 1: Random Walks. Barry D. Hughes, Clarendon Press, Oxford.

It is undoubtly somewhat questionable for me to review a book on random walks after having had one of my own published on this subject so short a time ago. It is a pleasure, however, for me te recommend the book. Hughes' book and mine overlap to some extent, but not entirely. Further, the two books are written in a somewhat different spirit, so that neither really preempts the other.

The present monograph covers a number of theoretical topics which I chose not to include, and does not contain the same variety of applications as is found in my own book. In contrast with my book, the present one tends to verge on the encyclopedic (hardly any number is quoted to fewer than six decimal places!), while my own was written more as an introductory text aimed at a graduate student in physics.

After the initial chapter summarizing some general elements of probability theory, the second chapter discusses, in exhaustive detail, the Pearson random walk, including an almost complete history of the origin of this early model. Included is an account of Lord Rayleigh's derivation of an exact solution, which, however, is not very useful in practice. The more convenient approach to deriving an exact solution in terms of Fourier series, originally suggested by Barakat, is referenced but not really discussed, and the arguably most significant application of the Pearson random walk and its multidimensional extensions to the interpretation of crystallographic data is only mentioned briefly as an aside.

A rather long and comprehensive chapter follows on properties of lattice random walks. This topic demonstrates the elegance of eighteenth and nineteenth century analytic methods in deriving a number of universal asymptotic properties of these walks. I found the discussion of taboo sites to be a little thin, but am possible overly influenced by a number of useful applications that have been made of the general idea in the theory of

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optical imaging by Amir Gandjbakhche and myself. An exhaustive discussion is given of the effects of defects and boundaries on properties of random walks.

The fourth chapter covers material on the transition between lattice random walks and the diffusion equation, as well as generalizations to Lévy flights. Also summarized is the very interesting work of Hughes, Shlesinger, and Montroll on self-similar random walks, i.e., walks whose jump probabilities are expressed in terms of the Weierstrass nondifferentiable function or other lacunary functions. The section on random walks with internal states gives a useful introduction to this topic but omits references to the important papers by Roerdinck and Shuler on the subject.

Chapter 5 discusses the continuous-time random walk (CTRW), which I am somewhat flattered to see referred to as the Montroll-Weiss random walk. Included is an account of the effective-medium approximation as well as an introduction to random walks on deterministic fractal lattices. This is followed by a chapter titled "Exploration and Trapping," which summarizes material on trapping models. This class of models has been extensively used to analyze a number of metallurgical problems as well as to provide a framework for discussing a variety of topics in the theory of chemical and other reaction rates. The trapping problem is a mathematically very demanding topic because it relates to non-Markovian properties, but is well worth pursuing because of the wide range of applications. Most of the discussion in this section is based on simpler aspects of the general theory, exemplified by exact solutions available in one dimension and the derivation of asymptotic results in higher dimensions using heuristic arguments given by Balagurov and Vaks and Grassberger and Procaccia. Extensions of these ideas to more general trapping models made by Berezhkovskii, Makhnovskii, and Suris are mentioned, as are variants of the trapping problem suggested by Stanley and his collaborators and Domb and Joyce.

The final chapter of the monograph presents introductory material on the self-avoiding random walk, first suggested in the 1950s as a more accurate model for the study of the configuration of polymer molecules than the standard random flight models in use until that time. A good deal of emphasis is placed on early approaches to finding the major properties of the self-avoiding walk as well as on what can be learned from rigorous arguments and specialized models. Wisely, the author has refrained from dealing in depth with the latest approaches to the problem, because such material will inevitably be obsolete in a very short time.

To summarize, this book contains an enormous amount of material about random walks in translationally invariant media in addition to an excellent bibliography of the research done in this general area up till 1994 with a slant toward physical applications. If the monograph can be faulted at all, it is that sometimes excessive detail obscures a major point. Nevertheless I recommend it as a scholarly and very competent piece of work, and certainly look forward to reading Volume 2, which is organized around the theme of transport in disorded media.

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